Group 1 : u-sub

#1. $\int \frac{z}{2z^2+8}\,dz$ $\quad u = 2z^2+8$

$du = 4z\,dz \Rightarrow \frac{du}{4} = z\,dz$

$= \int \frac{1}{u}\,\frac{du}{4}$

$= \frac{1}{4}\int \frac{1}{u}du = \frac{1}{4}\ln|u| + C = \frac{1}{4}\ln|2z^2+8| + C$

#2. $\int x e^{x^2-1}\,dx$ $\quad u = x^2-1$

$du = 2x\,dx \Rightarrow \frac{du}{2} = x\,dx$

$= \int e^u\,\frac{du}{2}$

$= \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2-1} + C$

#3. $\int \frac{1}{x(\ln x)^2}\,dx$ $\quad u = \ln x$

$du = \frac{1}{x}\,dx$

$= \int \frac{1}{u^2}\,du$

$= \int u^{-2}du = \frac{u^{-1}}{-1} + C = -(\ln x)^{-1} + C$

#4. $\int \frac{1}{x\ln(3x)}\,dx$ $\quad u = \ln(3x)$ ← chain rule!!

$du = \frac{1}{3x}\cdot 3\,dx = \frac{1}{x}\,dx$

$= \int \frac{1}{u}du$

$= \ln|u| + C = \ln|\ln(3x)| + C$

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hard!

#5. $\int (x^2+1)e^{-x}\,dx$ $\quad u = x^2+1$ $\quad v = \int e^{-x}dx$

$du = 2x\,dx$ $\quad = -e^{-x}$

$= (x^2+1)(-e^{-x}) - \int (-e^{-x})\,2x\,dx$

$= -(x^2+1)e^{-x} + 2\int e^{-x}\cdot x\,dx$ IBP again!!!

$u = x$ $\quad v = \int e^{-x}dx$

$du = dx$ $\quad = -e^{-x}$

$= -(x^2+1)e^{-x} + 2\left(-xe^{-x} - \int -e^{-x}dx\right)$

$= -(x^2+1)e^{-x} - 2xe^{-x} + 2\int e^{-x}dx$

$= -(x^2+1)e^{-x} - 2xe^{-x} + 2\frac{e^{-x}}{-1} + C$

Group 2 : IBP

#1. $\int \underbrace{x}_{u} \underbrace{e^{2x}}_{dv}\,dx$ $\quad u = x$ $\quad dv = e^{2x}dx$

$du = dx$ $\quad v = \int e^{2x}dx = \frac{e^{2x}}{2}$

$= \int u\,dv$

$= uv - \int v\,du$

$= x\cdot\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}dx$

$= x\frac{e^{2x}}{2} - \frac{1}{4}e^{2x} + C$

Remember, we did earlier

$\int e^{ax}dx = \frac{e^{ax}}{a} + C$

#2 $\int \frac{y}{e^{2y}}dy = \int \underbrace{y}_{u}e^{-2y}dy$

$u = y$ $\quad v = \int e^{-2y}dy$

$du = dy$ $\quad = \frac{e^{-2y}}{-2}$

$= uv - \int v\,du$

$= y\frac{e^{-2y}}{-2} - \int \frac{e^{-2y}}{-2}dy$

$= -\frac{1}{2}ye^{-2y} + \frac{1}{2}\int e^{-2y}dy$

$= -\frac{1}{2}ye^{-2y} - \frac{1}{4}e^{-2y} + C$

#3 $\int \underbrace{x^2}_{v}\underbrace{\ln x}_{u}\,dx$ $\quad u = \ln x$ $\quad v = \int x^2dx$

$du = \frac{1}{x}dx$ $\quad = \frac{x^3}{3}$

$= \frac{x^3}{3}\ln x - \int \underbrace{\frac{x^3}{3}}_{v}\cdot\underbrace{\frac{1}{x}dx}_{du}$

$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2dx$

$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$

#4. $\int x\underbrace{\ln(3x)}_{u}dx$ $\quad u = \ln(3x)$ $\quad v = \int x\,dx$

$du = \frac{1}{3x}\cdot 3dx = \frac{1}{x}dx$ $\quad = \frac{x^2}{2}$

$= \underbrace{\frac{x^2}{2}}_{v}\underbrace{\ln(3x)}_{u} - \int \underbrace{\frac{x^2}{2}}_{v}\cdot\underbrace{\frac{1}{x}dx}_{du}$

$= \frac{x^2}{2}\ln(3x) - \frac{1}{2}\int x\,dx$

$= \frac{x^2}{2}\ln(3x) - \frac{1}{4}x^2 + C$